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# Modification of the Blonder, Tinkham and Klapwijk theory of normal metal–superconductor point contact due to contact heterogeneity

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**Abstract.** A modification is proposed of the Blonder, Tinkham and Klapwijk (BTK) theory for heterogeneous normal metal–superconductor (N-s) point contact. Contact is heterogeneous when the normal state electronic dispersion relations in the left-hand and right-hand electrodes are different (in the original BTK theory dispersion relations in both electrodes are equal).

For characterization of the dispersion relation difference we use the amplitude of reflection  $r(E)$  at the clear interface for electrons incoming from the normal metal electrode with energy  $E_F + E$  when the 'superconductive' electrode is in the normal state. We calculate the amplitudes of ordinary and Andreev reflection at the N-s interface for electrons incoming from the normal metal electrode as functions of  $r(E)$ . This enables us to obtain an expression for differential conductance of the point contact as a function of applied voltage and  $r(eV)$ .

For a suitable function  $r(eV)$  the calculated differential conductance decreases with increasing voltage in the low-voltage region (as is observed experimentally for some point contacts between normal metal–high- $T_c$  superconductor) in contrast to the BTK theory which provides only non-decreasing dependences of differential conductance against voltage.

## 1. Introduction

Point contact spectroscopy is a useful tool for investigating the quasi-particle properties of solids. When applied to the classical superconductors it provides information on the energy gap and on the Eliashberg function of electron–phonon interaction.

The theory of Blonder, Tinkham and Klapwijk (BTK) [1] represents a suitable basis for interpretation of the point contact spectra (differential conductance versus voltage) if one electrode is normal metal and the second one is ordinary s-wave superconductor, at least for low voltage ( $eV \leq 2\Delta$ ).

But there are some experimental curves which cannot be explained within the framework of the BTK theory if a high  $T_c$  superconductor is used as the second electrode (for example in [2]). To interpret these measurements it is probably necessary either to devise a new theory (if superconductivity is not s-wave) or to modify the BTK theory if superconductivity is s-wave (as is probably the case in high- $T_c$  superconductors [3]). Our goal in this paper is to modify the BTK theory.

Blonder *et al* [1] obtained the expression for current through the ballistic normal metal–superconductor point contact in the form (for the one-dimensional case)

$$I = 2N(E_F)ev_F S \int_{-\infty}^{\infty} dE [f_0(E - eV) - f_0(E)][1 + A(E) - B(E)]. \quad (1)$$

Here  $N(E_F)$  refers to the one-spin electronic density of states at Fermi energy  $E_F$  in the normal electrode,  $S$  is a point contact cross-sectional area,  $v_F$  is the Fermi velocity of electrons in normal metal and  $f_0(E)$  is the equilibrium Fermi–Dirac distribution function.  $B(E)$  is the probability of the ordinary reflection at the N-S interface for electrons incoming from normal metal with energy  $E_F + E$  and  $A(E)$  is the probability of Andreev reflection.

In the real three-dimensional situation we will have

$$I \sim \int_{-\infty}^{\infty} dE [f_0(E - eV) - f_0(E)] \oint_{v_z > 0} \frac{dS(E)}{v_{\perp}} v_z [1 + A(E, \varphi) - B(E, \varphi)]$$

instead of expression (1) (see [1] and the method used for deriving the  $I$ – $V$  characteristic in [4]). Here  $\varphi$  is the angle of incidence of the particle to the interface;  $A(E, \varphi)$  and  $B(E, \varphi)$  are in general dependent on  $\varphi$ .  $\oint dS(E)$  is the integral over the equi-energy surface corresponding to energy  $E_F + E$  in  $k$ -space (in the normal metal electrode). Expanding the  $A(E, \varphi)$  and  $B(E, \varphi)$  into the Taylor series with respect to  $\sin^2 \varphi$ , integrating the members of series over equi-energy surface and taking into account that  $A(E, \varphi)$  and  $B(E, \varphi)$  are slowly varying functions of  $\varphi$  around  $\varphi = 0$ , it is possible to show that the main contribution to the current is provided by the first member of expansion, e.g. expression

$$\oint \frac{dS(E)}{v_{\perp}} v_z (1 + A(E, 0) - B(E, 0)).$$

So to study a qualitative behaviour of the  $I$ – $V$  characteristics of a point contact it is sufficient (and mathematically convenient) to deal with the one-dimensional geometry (the case of the BTK theory), e.g. to start with expression (1). Of course, if we are interested in quantitative values or if the investigated materials are strongly anisotropic, it is necessary to use the exact expression for current and to work in three-dimensional geometry. For the purpose of this paper it is sufficient to use the one-dimensional approximation (as in the original BTK theory).

To evaluate the quantities  $A(E)$  and  $B(E)$ , BTK used Bogoliubov equations which describe the excitations above the ground state of superconductor (for simplicity they solved the one-dimensional case)

$$\begin{aligned} H_0(x, \partial/\partial x)u(x) + \Delta(x)v(x) &= Eu(x) \\ -H_0(x, \partial/\partial x)v(x) + \Delta(x)u(x) &= Ev(x) \end{aligned} \quad (2)$$

where

$$H_0(x, \partial/\partial x) = (-\hbar^2/2m)\partial^2/\partial x^2 - E_F + V(x).$$

Here  $\Delta(x)$  is the energy gap of superconductor and  $V(x)$  is the one-particle potential.

In this formalism the two-element column vector formed from  $u(x)$  and  $v(x)$  can be interpreted as a two-component excitation wave function (in a superconductor it represents a combination of particle and hole).

To model the effect of any interfacial elastic scattering BTK [1] included in equation (2) a  $\delta$ -function repulsive potential located at the interface in the form  $V(x) = H\delta(x)$ . Quantity  $H$  (or  $Z = mH/\hbar^2 k_F$ ) characterizes the barrier strength at the interface (oxide

layer, localized disorder in contact, etc.). The change of  $Z$  enables one to model the crossover from metallic ( $Z = 0$ ) to tunnel junction behaviour ( $Z \geq 10$ ) of the contact.

The BTK solution of equation (2) for the normal electrode is  $f_{\text{inc}} + f_{\text{refl}}$  where

$$f_{\text{inc}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp \left\{ \frac{i}{\hbar} [2m(E_F + E)]^{1/2} x \right\}$$

$$f_{\text{refl}} = a(E) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp \left\{ \frac{i}{\hbar} [2m(E_F - E)]^{1/2} x \right\} + b(E) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp \left\{ -\frac{i}{\hbar} [2m(E_F + E)]^{1/2} x \right\}.$$

The quantities  $A(E)$  and  $B(E)$  required in (1), are defined as  $A(E) = a^*(E)a(E)$  and  $B(E) = b^*(E)b(E)$ . In paper [1] the concrete forms of  $a(E)$  and  $b(E)$  were obtained by applying suitable boundary conditions to the solutions of equation (2) for left-hand and right-hand electrodes. The BTK result is

$$a(E) = \Delta / [E + (1 + 2Z^2)(E^2 - \Delta^2)^{1/2}] \quad (3)$$

$$b(E) = [-2(Z^2 + iZ)(E^2 - \Delta^2)^{1/2} / [E + (1 + 2Z^2)(E^2 - \Delta^2)^{1/2}]]. \quad (4)$$

Using (3), (4) and (1) Blonder *et al* [1] obtained the expressions for current and for the differential conductance  $dI/dV$  of the contact. Their result is that differential conductance is constant (for  $Z = 0$ ) or increases (for  $Z > 0$ ) with increasing voltage for  $eV \leq \Delta$ .

Among the measured spectra of  $dI/dV$  against  $V$  for a high- $T_c$  superconductor there are curves decreasing with increasing voltage for low voltage. Such non-standard characteristics are obtained especially for low-resistance pressure point contacts [2].

In paper [5] we explained some features of these anomalous curves by the occurrence of a Josephson junction inside the superconductor near the contact. But also in this case the differential conductance around zero bias is determined only by the point contact normal metal–superconductor alone. So it is necessary to modify the BTK theory.

In the BTK theory it is assumed that the Hamiltonian  $H_0$  in (2) is the same in both electrodes and the normal electrode is distinguished from the superconducting one only by putting  $\Delta = 0$ . Our modification of the BTK theory consists of changing the form of quantities  $A(E)$  and  $B(E)$  in the expression (1), due to the fact that Hamiltonian  $H_0$  is, in general, different in the left-hand and right-hand electrodes.

In our treatment a quantity which characterizes the difference between Hamiltonians  $H_0$  for different electrodes is an amplitude of reflection  $r(E)$  at the clear interface for the electron incoming from normal metal with energy  $E_F + E$  if both electrodes are in the normal state.

We derived the modified amplitudes  $a(E)$  and  $b(E)$  as functions of  $\Delta$ ,  $Z$  and  $r(E)$  for the one-dimensional case. Using (1) and our results  $a(E)$  and  $b(E)$  we derived the expression for differential conductance as a function of  $r(eV)$  for low temperature. For small  $Z$  (clear interface), differential conductance is determined by  $r(eV)$  and  $r(-eV)$  and it can be a decreasing function of voltage.

## 2. Modification of the BTK theory

The expression (1) for current through ballistic point contact is general but quantities  $A(E)$  and  $B(E)$  are dependent on a model chosen for normal metal and a superconductor.

For calculation of  $A(E) = a^*(E)a(E)$  and  $B(E) = b^*(E)b(E)$  we use Bogoliubov equations (for the one-dimensional case) describing an excitation in a N-S (in general heterogeneous) system

$$\mathbf{H}(x, \partial/\partial x)\mathbf{f}(x) = E\mathbf{f}(x) \quad (5)$$

$$\mathbf{H}(x, \partial/\partial x) = [(1 - \Theta(x))\varepsilon_L(\partial/\partial x) + \Theta(x)\varepsilon_R(\partial/\partial x) + \delta(x)H]\boldsymbol{\tau}_3 + \Theta(x)\Delta\boldsymbol{\tau}_1.$$

Here  $\boldsymbol{\tau}_i$  are Pauli matrices:

$$\boldsymbol{\tau}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \boldsymbol{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$\Theta(x) = 1$  for  $x \geq 0$  and  $\Theta(x) = 0$  for  $x < 0$  and  $\varepsilon_{L(R)}(\partial/\partial x) \exp(ikx) = (\varepsilon_{L(R)}(\mathbf{k}) - E_F) \exp(ikx)$ .  $\varepsilon_{L(R)}(\mathbf{k})$  is normal-state electronic dispersion relation in the left-hand (right-hand) electrode. Member  $\delta(x)H$  in (5) represents (as in the BTK theory) elastic repulsion at N-S interface (due to an oxide layer, deformations, etc.).  $\mathbf{f}(x)$  is the two-component excitation wave function.

We are interested in the solution of equation (5) having the form  $\mathbf{f}(x) = \mathbf{f}_{\text{inc}}(x) + \mathbf{f}_{\text{refl}}(x)$  for  $x < 0$ :

$$\mathbf{f}_{\text{inc}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(ik_p x) \quad \mathbf{f}_{\text{refl}} = a(E) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(ik_h x) + b(E) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(-ik_p x). \quad (6)$$

We assume that particle group velocity direction is identical to the direction of  $\mathbf{k}$ . Subscript p or h determines the kind of excitation (particle or hole). Wavevectors  $k_p$  and  $k_h$  as functions of  $E$  are determined by equations  $\varepsilon_L(k_p) = E_F + E$  and  $\varepsilon_L(k_h) = E_F - E$ .

We suppose that we are able to solve equation (5) for  $\Delta = 0$ , e.g. that amplitudes  $r$  and  $t$  in the following expressions are known (the first subscript determines the kind of incoming excitations and the second one determines the direction of its group velocity—direction p is from left to right and direction n is opposite).

$$\mathbf{f}_1 = (1 - \Theta(x)) \begin{pmatrix} 1 \\ 0 \end{pmatrix} [\exp(ik_p x) + r_{pp} \exp(-k_p x)] + \Theta(x) \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_{pp} \exp(iq_p x)$$

$$\mathbf{f}_2 = (1 - \Theta(x)) \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_{pn} \left[ \exp(-ik_p x) + \Theta(x) \begin{pmatrix} 1 \\ 0 \end{pmatrix} (\exp(-iq_p x) + r_{pn} \exp(iq_p x)) \right] \quad (7)$$

$$\mathbf{f}_3 = (1 - \Theta(x)) \begin{pmatrix} 0 \\ 1 \end{pmatrix} t_{hn} (\exp(ik_h x) + \Theta(x) \begin{pmatrix} 0 \\ 1 \end{pmatrix} [\exp(iq_h x) + r_{hn} \exp(-iq_h x)]).$$

$q_p$  and  $q_h$  are determined by equations

$$\varepsilon_R(q_p) = E_F + E \quad \text{and} \quad \varepsilon_R(q_h) = E_F - E.$$

Another assumption is that the solutions of equation

$$[\varepsilon_R(\partial/\partial x)\boldsymbol{\tau}_3 + \Theta(x)\Delta\boldsymbol{\tau}_1]\mathbf{f}(x) = E\mathbf{f}(x) \quad (8)$$

are known for  $x < 0$  in the form

$$\mathbf{f}_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} [\exp(iq_p x) + b_{pp}^0 \exp(-iq_p x)] + a_{pp}^0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(iq_h x) \quad (9)$$

$$\mathbf{f}_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} [\exp(-iq_h x) + b_{hp}^0 \exp(iq_h x)] + a_{hp}^0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(-iq_p x)$$

(this corresponds to the case of homogeneous contact).

The goal is to express the amplitudes  $a(E)$  and  $b(E)$  from (6) by means of the amplitudes  $r$ ,  $t$ ,  $a^0$  and  $b^0$ .

Writing

$$f = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(ik_p x) + f_R(x)$$

and taking into account that

$$\varepsilon_L(\partial/\partial x)\tau_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(ik_p x) = E\mathbf{1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(ik_p x)$$

it is possible to rewrite equation (5) in the form

$$f_R(x) = \int_{-\infty}^{\infty} dx' \mathbf{G}^+(E, x, x') \left[ \mathbf{H} \left( x', \frac{\partial}{\partial x'} \right) - \varepsilon_L \left( \frac{\partial}{\partial x'} \right) \tau_3 \right] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(ik_p x'). \quad (10)$$

Here  $\mathbf{G}^+(E, x, x')$  is the Green's function of the linear operator occurring on the left-hand side of equation (5):

$$[(E + i0)\mathbf{1} - \mathbf{H}(x, \partial/\partial x)]\mathbf{G}^+(E, x, x') = \delta(x - x')\mathbf{1}.$$

For  $x < 0$

$$f_R(x) = f_{\text{refl}}(x).$$

Using in sequence different variants of the equation for Green's function (analogous to the equation (10) for the wave function and obtained by direct rewriting of the differential equation for Green's function into the integro-differential form) we get

$$\mathbf{G}^+(E, x, x') = \mathbf{G}_{\text{H1}}^+(E, x, x') + \int_{-\infty}^{\infty} dy \mathbf{G}^+(E, x, y) \mathbf{H}_2 \left( y, \frac{\partial}{\partial y} \right) \mathbf{G}_{\text{H1}}^+(E, y, x')$$

where

$$\mathbf{H}_1(x, \partial/\partial x) + \mathbf{H}_2(x, \partial/\partial x) = \mathbf{H}(x, \partial/\partial x).$$

$\mathbf{H}_1(x, \partial/\partial x)$  is a part of the Hamiltonian  $\mathbf{H}(x, \partial/\partial x)$  suitably chosen according to the situation investigated—equal or different normal-state dispersion relations in both electrodes, zero or non-zero barrier  $H\delta(x)$ , etc.  $\mathbf{H}_2(x, \partial/\partial x)$  is the rest of Hamiltonian  $\mathbf{H}(x, \partial/\partial x)$ , and

$$[(E + i0)\mathbf{1} - \mathbf{H}_1(x, \partial/\partial x)]\mathbf{G}_{\text{H1}}^+(E, x, x') = \delta(x - x')\mathbf{1}.$$

Taking into account equations (7), (9) and (10) it is possible to transform the right-hand side of equation (10) into a form containing only linear combinations of functions

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(-ik_p x) \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(ik_h x)$$

for  $x < 0$ . It is not necessary to derive the explicit mathematical forms of  $\mathbf{G}^+(E, x, x')$  and  $\mathbf{G}_{\text{H1}}^+(E, x, x')$  because only a formal decomposition of  $\mathbf{G}^+(E, x, x')$  by means of  $\mathbf{G}_{\text{H1}}^+(E, x, x')$  and  $\mathbf{H}_2(x, \partial/\partial x)$  is used and validity of expressions (7), (9) and (10) is

taken into account during this transformation. From this transformed equation we obtained the following relations among amplitudes:

$$a = \frac{t_{pp} a_{pp}^0 t_{hn}}{1 - b_{hp}^0 r_{hn} - b_{pp}^0 r_{pn} + r_{pn} r_{hn} (b_{pp}^0 b_{hp}^0 - a_{pp}^0 a_{hp}^0)} \quad (11)$$

$$b = r_{pp} + \frac{t_{pp} [b_{pp}^0 + r_{hn} (a_{pp}^0 a_{hp}^0 - b_{pp}^0 b_{hp}^0)] t_{pn}}{1 - b_{hp}^0 r_{hn} - b_{pp}^0 r_{pn} + r_{pn} r_{hn} (b_{pp}^0 b_{hp}^0 - a_{pp}^0 a_{hp}^0)} \quad (12)$$

and  $t_i = 1 + r_i$ .

By similar means to  $a$  and  $b$  it is possible to express the full amplitudes of reflection  $r_{pp}$  and  $r_{pn}$  by means of the amplitudes of reflection at the  $\delta$ -function potential for homogeneous contact (where the right-hand electrode is made of the same material as the left-hand one) and amplitudes of reflection at clear interface  $r^i$ . The results are

$$r_{pp} = r_{pp}^\delta + t_{pp}^\delta [r_{pp}^i / (1 - r_{pp}^i r_{ph}^\delta)] t_{pn}^\delta \quad (13)$$

$$r_{pn} = r_{pn}^i + t_{pn}^i [r_{pn}^\delta / (1 - r_{pn}^\delta r_{pp}^i)] t_{pp}^i. \quad (14)$$

If the left-hand electrode is made from ordinary metal (Cu, Ag, Au, etc) then

$$r_{pp}^\delta = r_{pn}^\delta = -iZ(1 + iZ)^{-1} \quad (15)$$

and  $Z = mH/\hbar^2 k_F$  (as in the BTK theory).

Amplitudes  $a_{pp}^0$ ,  $b_{pp}^0$ ,  $a_{hp}^0$ ,  $b_{hp}^0$  are not mutually independent. Using the fact that functions (9) are the solutions of equation (8) and performing some simple transformations of equation (8) it is possible to show that

$$y_{hp}(E, \Delta, (E^2 - \Delta^2)^{1/2}) = [y_{pp}(-E, -\Delta, -(E^2 - \Delta^2)^{1/2})]^* \quad (16)$$

where  $y = a^0$  or  $y = b^0$ . By similar means it is possible to show that

$$r_{hn}(E) = [r_{pn}(-E)]^*. \quad (17)$$

If  $\varepsilon_L(\mathbf{k})$  and  $\varepsilon_R(\mathbf{q})$  are even functions of  $\mathbf{k}$  and  $\mathbf{q}$  then

$$r_{pp}^i(E) = -r_{pn}^i(E) = r(E) \quad (18)$$

and  $r(E)$  is a real function if  $E_F \pm E$  lie in the conduction bands in both electrodes.

Now we assume that the left-hand electrode is ordinary metal,  $\varepsilon_R(\mathbf{q})$  is an even function of  $\mathbf{q}$  and is a smooth function of  $\mathbf{q}$  around  $E_F$ . So we can use for  $a_{pp}^0$ ,  $b_{pp}^0$  the BTK results for  $Z = 0$ . From (3) and (4) we have

$$b_{pp}^0 = 0 = b_{hp}^0 \quad a_{pp}^0 = \Delta[E + (E^2 - \Delta^2)^{1/2}]^{-1} = a_{hp}^0.$$

Finally we obtain for  $a$  and  $b$  (using (11)–(18))

$$a = \Delta(1 + r(E))(1 - r(-E))D^{-1} \quad (19)$$

$$b = E(r(E) - r(-E))(1 - 2iZ)D^{-1} + [r(E) + r(-E) - 2(Z^2 + iZ)(1 + r(E))(1 + r(-E))]D^{-1} \quad (20)$$

and

$$D = E[1 - r(E)r(-E) + i2Z(r(E) - r(-E))] + [1 + r(E)r(-E) + 2Z^2(1 + r(E))(1 + r(-E))](E^2 - \Delta^2)^{1/2}.$$

For expression  $T(E) = 1 + aa^* - b^*b$  in equation (1) we have for  $E < \Delta$

$$T(E) = N(E < \Delta)/D(E < \Delta) \quad (21)$$

$$N(E < \Delta) = 2\Delta^2(1 + r(E))(1 - r(-E))(1 - r(E)r(-E))$$

$$D(E < \Delta) = E^2(1 - r(E)r(-E))^2 + [2ZE(r(E) - r(-E)) + X(E)]^2$$

$$X(E) = [1 + r(E)r(-E) + 2Z^2(1 + r(E))(1 + r(-E))](\Delta^2 - E^2)^{1/2}$$

for  $E > \Delta$

$$T(E) = N(E > \Delta)/D(E > \Delta) \quad (22)$$

$$N(E > \Delta) = 2(1 - r^2(E))E^2(1 - r^2(-E)) + 2(1 - r^2(E))$$

$$\times E[1 + r^2(-E) + 2Z^2(1 + r(-E))^2](E^2 - \Delta^2)^{1/2} + 2\Delta^2(1 + r(E))$$

$$\times (1 - r(-E))(r(E) - r(-E))$$

$$D(E > \Delta) = [E(1 - r(E)r(-E)) + X'(E)]^2 + 4E^2Z^2(r(E) - r(-E))^2$$

$$X'(E) = [1 + r(E)r(-E) + 2Z^2(1 + r(E))(1 + r(-E))](E^2 - \Delta^2)^{1/2}.$$

Finally, the measured quantity (differential conductance  $dI/dV$ ) has (for low temperature) the form

$$dI/dV = G_D(eV) = G_{DN}(0) \frac{1 + Z^2(1 + r(0))^2}{1 - r^2(0)} T(eV) \quad (23)$$

where  $G_{DN}(0)$  is the differential conductance for the same point contact but with both electrodes in the normal state and for zero bias

$$G_{DN}(0) = 2N(E_F)e^2v_F S(1 - r^2(0))/[1 + Z^2(1 + r(0))^2].$$

### 3. Discussion and conclusion

We derived the amplitudes of ordinary ( $b$ ) and Andreev ( $a$ ) reflection at a N-S interface with explicitly expressed contributions due to the superconductivity of the right-hand electrode (amplitudes  $a^0$ ,  $b^0$ ), due to the repulsive potential at interface (quantity  $Z$ ) and due to the different dispersion relations in the left-hand and right-hand electrodes (amplitudes  $r^i$ ,  $r^i$ ). As a measure of the difference in the dispersion relations we used the amplitude of reflection  $r(E)$  at the clear interface for electrons incoming from the left-hand electrode if both electrodes are in the normal state.

If the BTK results are applicable for the amplitudes  $a^0$ ,  $b^0$  (where the left-hand electrode is from ordinary metal and the electronic dispersion relation in the right-hand electrode is a smooth function of wavevector around the Fermi energy) further simplification of the expressions for  $a$  and  $b$  is possible. We obtained amplitudes  $a$  (19) and  $b$  (20) as functions of the barrier strength  $Z$  at the interface (oxide layer, deformations, etc) and only one amplitude of reflection at clear interface  $r(E)$ . With this in hand we calculated the differential conductance of the point contact (23).

Function  $r(E)$  represents a model parameter in our results. If some model electronic dispersion relation for superconductor is chosen it is possible to calculate  $r(E)$  and then, according to (23), to evaluate the differential conductance for point contact normal



metal-chosen superconductor. When  $r(E) = 0$  in the relevant energy interval our relations (19) and (20) take the BTK form (3) and (4).

For large voltage  $eV \gg \Delta$  coefficient  $T(eV)$ , defined by (22), approaches the electronic coefficient of transmission through the interface for both electrodes in the normal state. Differential conductance for low temperature is proportional to the  $T(eV)$  so it can serve for large  $eV$  and clear interface (small  $Z$ ) for investigation of the normal-state dispersion relation in the 'superconducting' electrode (due to the dependence of  $r(E)$  on dispersion relation).

For small voltage, expression (21) takes the form

$$T(eV) \sim \frac{2(1 - r^2(0))}{[1 + r^2(0) + 2Z^2(1 + r(0))^2]^2} (1 - r^2(0) + r(eV) - r(-eV)).$$

So differential conductance around zero bias is determined by the processes of transmission of electrons and holes through the interface. If  $r(E)$  is suitably chosen  $dI/dV$  can be a decreasing function of voltage for small  $V$  (in contrast to the original BTK theory).

We showed that the difference in the electronic dispersion relations of the electrodes in the N-S point contact affects the differential conductance.  $dI/dV$  can also be a decreasing function of  $V$ . The influence of  $r(E)$  decreases as  $Z$  increases. For large  $Z$  the dominant contribution to the  $r_{pp}$ ,  $r_{pn}$  and  $r_{hn}$  is due to the energy-independent quantity  $Z$ . So it is possible to observe the manifestation of the normal state electronic dispersion relations in point contacts with clear interface (small  $Z$ ). In contacts with large  $Z$  (classical tunnelling junctions) this phenomenon is suppressed.

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